Polynomials and Like Terms

Definition of a Polynomial

Algebraic expressions that are numbers, powers of variables, or products of numbers and powers of variables are called **terms**. That is, an expression that involves only multiplication and/or division with variables and constants is called a **term**. Remember that a number written next to a variable indicates multiplication, and the number is called the **coefficient** of the variable. For example,

Algebraic terms in one variable:

$5x$, $25x^3$, $6y^4$, and $48n^2$

Algebraic terms in two variables:

$3xy$, $4ab$, $6xy^5$, and $10x^2y^2$

A term that consists of only a number, such as 2 or −7, is called a **constant** or a **constant term**. In this text we will discuss terms in only one variable and constants.

In algebra, we are particularly interested in the study of expressions such as

$5x + 7$, $3x^2 - 2x + 4$, and $6a^4 + 8a^3 - 10a$

that involve sums and/or differences of terms. These algebraic expressions are called **polynomials**.

Polynomial: A **polynomial** is a monomial or the indicated sum or difference of monomials.

The **degree of a polynomial** is the largest of the degrees of its terms.

Example 1: Degree of a Polynomial

Determine the degree of each of the following polynomials.

a. $6x - 3$ **b.** $x^5 + 2x^3 - 16$ **c.** $7y^{17}$

Solutions:

a. The polynomial $6x - 3$ can be written as $6x^1 - 3x^0$. The term with the largest degree is $6x^1$. Since the degree of the term is 1, the degree of $6x - 3$ is 1.

b. The polynomial $x^5 + 2x^3 - 16$ can be written as $x^5 + 2x^3 - 16x^0$. The term with the largest degree is x^5. Since the degree of the term is 5, the degree of $x^5 + 2x^3 - 16$ is 5.

c. The polynomial $7y^{17}$ only has one term. Since the degree of the term is 17, the degree of $7y^{17}$ is 17.

Some forms of polynomials are used so frequently that they have been given special names, as indicated in the following box.

Classification of Polynomials:

Description	Name	Example
Polynomial with one term	**Monomial**	$5x^3$
Polynomial with two terms	**Binomial**	$7x - 23$
Polynomial with three terms	**Trinomial**	$a^2 + 5a + 6$

Example 3: Classification of Polynomials

Name the type of each of the following polynomials.

a. $2x^4 - 7x^2 + 13$ **b.** $3y^{23}$ **c.** $a - 22$

Solutions:

a. $2x^4 - 7x^2 + 13$ is a trinomial.

b. $3y^{23}$ is a monomial.

c. $a - 22$ is a binomial.

$-1, 14, 256$	are like terms because each term is a constant.
$-13a, 9a, 19a$	are like terms because each term contains the same variable a, raised to the same power, 1.
$-14x^2y$ and $9x^2y$	are like terms because each term contains the same two variables, x and y, where x is second-degree in both terms and y is first-degree in both terms.

Unlike Terms

$7x$ and $8x^2$	are unlike terms (**not** like terms) because the variable x is not of the same power in both terms.
$6ab^2$ and $2a^2b$	are **not** like terms because the variables are not of the same power in both terms.

If no number is written next to a variable, then the coefficient is understood to be 1. For example,

$$x = 1 \cdot x, \quad a^3 = 1 \cdot a^3, \quad \text{and} \quad xy = 1 \cdot xy.$$

If a negative sign (–) is written next to a variable, then the coefficient is understood to be –1. For example,

$$-y = -1 \cdot y, \quad -x^2 = -1 \cdot x^2, \quad \text{and} \quad -xy = -1 \cdot xy.$$