Solving systems of equations by graphing

When two linear equations are consider together, they are called a system of linear equations. The term simultaneous is frequently used to emphasize the idea that the solution of a system is the point that satisfies both equations at the same time, or simultaneously.

If a system has a unique solution (one point of intersection), the system is **consistent**. If a system has no solution (the lines are parallel with no point of intersection), the system is **inconsistent**. If a system has an infinite number of solutions (the lines coincide), the system is **dependent**.

The following table and graphs summarize the basic ideas and terminology.

System	Graph	Intersection	Terms
$3x + y = 5$ $x = 2y - 3$	Graphically	One point: (1, 2)	**Consistent**
$y = -x + 4$ $y = -x + 2$	Graphically	No points; lines are parallel	**Inconsistent**
$y = -x + 4$ $2x + 2y = 8$	Graphically	Infinite number of points; lines are the same (they coincide)	**Dependent**

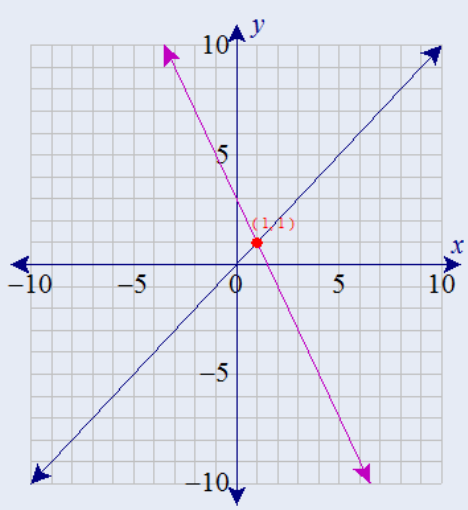
Determine graphically whether the following systems are consistent, inconsistent, or dependent. If the system is consistent, find (or estimate) the point of intersection.

a. $x - y = 0$
$2x + y = 3$

Solution:

Solve each equation for y, writing the equations in slope-intercept form, and graph on the same xy-axis.

The point of intersection is (1, 1), so the system is **consistent**.



Check:

Substitute $x = 1$ and $y = 1$ into both equations.

For $x - y = 0$, we have $1 - 1 = 0$.

For $2x + y = 3$, we have $2 \cdot 1 + 1 = 3$.

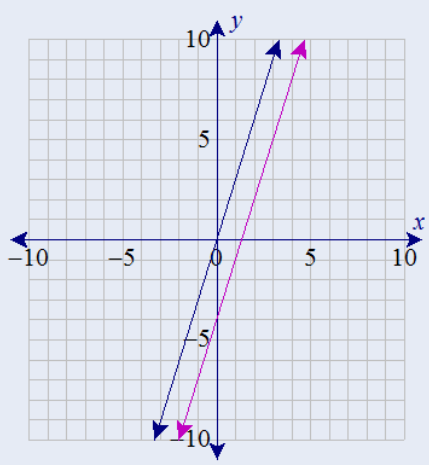
When a system of equations is **inconsistent**, both equations have the ***same slope and different "y" intercept***

Solving for "y" both equations we have: y = 3x + 0

y = 3x + (- 4)

c. $\begin{aligned} y &= 3x \\ y - 3x &= -4 \end{aligned}$

Solution: Solve each equation for y, writing the equations in slope-intercept form, and graph on the same xy-axis. The system is **inconsistent**. The lines are parallel with the same slope, 3, and there are no points of intersection. Thus there is no solution, Ø.



When a system of equations is **dependent,** when solving for "y" we have ***the same line***

Solving for "y" in both equations we have: x + 2y = 6 2y = -x + 6 $y = -x/2 + 3$

$y = -x/2 + 3$

d. $\begin{aligned} x + 2y &= 6 \\ y &= -\frac{1}{2}x + 3 \end{aligned}$

Solution: Solve each equation for y, writing the equations in slope-intercept form, and graph on the same xy-axis. The system is **dependent**. All points that lie on one line also lie on the other line.

For example, $(4, 1)$ is a point on the line $x + 2y = 6$ since $4 + 2(1) = 6$.

The point $(4, 1)$ is also on the line $y = -\frac{1}{2}x + 3$ since $1 = -\frac{1}{2}(4) + 3$.

Writing both equations in slope-intercept form gives $y = -\frac{1}{2}x + 3$. Thus the solution consists of all points that satisfy this equation. We can write the solution in the general form $\left(x, -\frac{1}{2}x + 3\right)$.